Application of Computational Limit Analysis & Design in the Middle East

Matthew Gilbert
BEng PhD CEng MICE MASCE
Professor at the University of Sheffield &
Managing Director at LimitState Ltd
Contents

1. Background
2. Discontinuity Layout Optimization (DLO)
3. Application in design
4. Middle East examples
5. Conclusions
Background
Available geotechnical software

‘Traditional’: based on hand analysis solutions etc.

‘Advanced’: based on non-linear finite elements etc. (potentially embedded in simple programs / spreadsheets etc.)

Gap…

More:
• complex
• time consuming
• input parameters
• expertise required
• accurate [potentially at least!]
‘Traditional’ example

- Bearing capacity
  - Terzaghi equation:

\[
q = cs_{c} d_{c} i_{c} N_{c} + qs_{q} d_{q} i_{q} N_{q} + \frac{1}{2} \gamma B s_{\gamma} d_{\gamma} i_{\gamma} N_{\gamma}
\]

[Approximate as terms in equation are not really additive – though addition always gives conservative result]

Issue: to handle embedded foundations, inclined loads, layered soils etc. it is necessary to apply additional modification factors. This becomes increasingly imprecise as more factors are used.
The role of limit analysis

- Many ‘traditional’ methods use limit analysis
- Limit analysis allows direct estimation of the maximum load sustainable by a body or structure

**Question:** can limit analysis be computerised?
Available geotechnical software

‘Traditional’: based on hand analysis solutions etc. (potentially embedded in simple programs / spreadsheets etc.)

‘Mainstream’: based on computational limit analysis(?)

‘Advanced’: based on non-linear finite elements etc.

More:
• complex
• time consuming
• input parameters
• expertise required
• accurate [potentially at least!]
Computational limit analysis

• Automated single application methods, e.g:
  – Foundations: slip-line field analysis
  – Walls: Coulomb wedge analysis

• Numerical methods
  – Method of characteristics
    • e.g. ABC software for foundations
  – Finite element limit analysis
    • Research tool only at present
  – Discontinuity Layout Optimization (DLO)
    • Now widely used in industry
Footings

Slopes

Retaining walls

‘Combined’

Soil-structure interaction
Discontinuity Layout Optimization (DLO)
Discontinuity Layout Optimization (DLO)

- Can be used to provide rigorous upper bound limit analysis solutions:
  - rapid and direct means of analysing the collapse state
  - outcome of 5 year EPSRC (UK Government) funded research project carried out at the University of Sheffield
DLO: problem formulation

• Upper bound (‘mechanism’) formulation
  – Find minimum multiplier on specified loads (margin of safety, or ‘adequacy factor’)
  – Subject to nodal compatibility constraints:

\[
\sum_{i=1}^{5} s_i \cos \theta_i = 0
\]
\[
\sum_{i=1}^{5} s_i \sin \theta_i = 0
\]
DLO: obtaining a solution

- Problem is a simple linear optimization problem (easy to solve)
- After solving can identify and then deform solid blocks to help interpretation:
Application to benchmark problems

• For ‘Prandtl punch’ problem, solution within 1% of exact solution ($2+\pi$) in approx. 1 second:

• Results for 100+ other cohesive-frictional benchmark plane strain problems available at: http://www.limitstate.com/geo/verification
Application examples

Inclined footing, carrying stadium roof loads

Trench excavation near footing
Application examples

- Quay wall
- Mast foundation
- Slope remediation
Application examples

- Masonry arch stability
- Anchored sheet pile wall
- Construction on slope
Application in practice

- Once embedded in a modern software application, methods such as DLO takes the work out of analysis and allow the engineer to focus on:
  - Interpreting and understanding the ground conditions
  - Developing innovative designs
  - What if/parametric studies
Recent academic developments

- Curved slip-lines to model rotational failures
  - Smith & Gilbert, *Géotechnique*, November 2013

- 3D DLO
Application in design
Application in design

• DLO provides engineers with a fully general-purpose direct ULS analysis capability.

• How can this be used with existing and new design codes, including limit state codes such as Eurocode 7?
Slope stability

Conventional approach:
• Factor on strength averaged over length of assumed slip surface.

General approach:
• Factor on strength applied to whole soil domain. Software automatically determines critical failure mechanism.
Retaining walls

Conventional approach:
• Active and passive earth pressures are **assumed** to act on the wall. $K_a$ and $K_p$ determined from factored soil strength.

General approach:
• Factor on strength applied to whole soil domain. Software **automatically determines** critical failure mechanism.
Conventional approach:
• Determine collapse load using Terzaghi’s bearing capacity equation (implicitly assuming a specific collapse mode). Factor load by 3.

General approach:
• Determine collapse load from automatically determined critical failure mechanism. Factor load by 3.
What about limit state design?

- Limit state design codes such as Eurocode 7 (EC7) are written in a general way, often tying in with general purpose analysis approaches.

- e.g. EC7 Design Approach 1 (DA1) Combination 2 allows direct use of general purpose numerical limit analysis.
What about limit state design? [2]

• Before undertaking an analysis **partial factors** are applied to loads and/or materials.

• An **adequacy factor** is then applied to load(s) or material strength(s) to trigger collapse:
  – If the **adequacy factor** $\geq 1.0$ then the system is stable.
  – If the **adequacy factor** $< 1.0$ then the system is unstable.
Middle East examples
Example 1: embankment on soft soil

- Soft soils are common in the region:

(Photos from Benmebarek et al., TAJSAT, 2013)
‘Traditional’ model

- Traditional methods of analysis rely on assumed mechanisms
- e.g. Hird (1986):
LimitState:GEO model

- Key point: able to identify a wider range of mechanisms than traditional methods, e.g:
Example 2: foundation close to cavity

- Dissolution cavities common in the region

- Problem: difficult to model these using traditional methods!
Example 2: foundation close to cavity
LimitState:GEO model

• Easy to model effects of arbitrary features, such as voids

(assuming their location & extent are known!)
Other applications

• Modelling of soil improvement methods
  – Cut and fill
  – Stone columns
  – Ground mixing
  – Etc, etc…
Conclusions
Conclusions

• Computational limit analysis fills the gap between ‘traditional’ and ‘advanced’ tools:
  – Generally applicable methods fit in well with the philosophy of limit state codes such as Eurocode 7

• DLO is a quick and easy to use general method:
  – Automatically identifies the critical mechanism
  – Eliminates the need to separately consider sliding, bearing failure etc, or to try to fit a problem to a known solution

• Flexibility to model non-standard features / geometries is important for many Middle East applications
Acknowledgements

• LimitState credits:
  – Dr Wael Darwich, Dr Tom Pritchard

• University of Sheffield credits:
  – Dr Colin Smith, Dr Sam Hawksbee, Alireza Tatari, Ahmed Babiker
  – EPSRC (UK Government Research Council)